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13. ABSTRACT (Maximum 200 words)

The aim of the project was to investigate features of binary images by representing them in terms of polynomials in two variables. We developed an algebraic environment to process such images and further extended our algebraic approach to colored images and 3D images in voxel representation. Our algorithms are very efficient and mostly have linear run-times in the number of pixels. Besides several technical papers written under this Grant, one Ph.D. and one M.S. was completed by the research assistants working on this project.

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STUDY OF FEATURES OF BINARY IMAGES USING IMAGE ALGEBRA TECHNIQUES

Final Technical Report: Grant No. AFOSR-90-0046

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1 OBJECTIVES OF THE PROJECT

The aim of this project was to investigate features of binary images by considering a special case of the Image Algebra methodology obtained by representing digitized images (both monochrome and colored) by certain polynomials in two variables with coefficients from the binary field. Since polynomials can be easily manipulated and our proposed operators can be described conveniently in terms of algebraic operations on these polynomials, this approach provides a significant foundation for practical applications which would be of significant interest to AFOSR.

Our specific objectives have been as follows. We have developed algebraic operators in the context of the polynomial approach to determine the contour, magnification and shrinking, and a sequence of approximations (from finer to coarser) of a binary image. Further, we have extended our techniques to process gray images and do operations such as template decomposition, shape decomposition, connected component labelling. Also, we have developed an algebraic system to process colored images. Also, we have developed some fast sequential and parallel thinning algorithms. We have also extended the polynomial approach to 3-D by developing equivalents of the standard morphological operations and applying these to do a number of operations for the understanding of 3D objects.

Besides several technical papers prepared under this Grant (see Section 8 for the complete list) this Grant also supported the successful completion of **one Ph.D. thesis and one M.S. thesis** under the supervision of the Principal Investigator.

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2 OUR WORK ON BINARY IMAGES

A Contour Operator. Agui et al (1982) had defined an operator given by a polynomial called the *differentiation* operator which when operated on the polynomial representing a binary image A gives allegedly the contour of the image. Also, they give two other operators which are purported to give the contour of the image. We showed by giving examples that these contour operators proposed by them for binary pictures are far from satisfactory. Consequently, we developed a refinement of their contour operators by proposing a Boolean algebra with new definitions of addition and multiplication and defining a new contour operator in this setting. We have tested our operator on a number of binary images and have found it to give a much better realization of the contour

A-1

as compared to what one gets by applying the operators described by Agui et al. The **time complexity** of our contour algorithm is O(n) where n is the number of black pixels in the binary picture.

Approximation. It is of practical interest to construct an approximation of a binary picture. First, we observe that since we have devised an efficient data structure in the form of a *Quadtree* , it is natural to proceed to approximate a digitized image following the quadtree. By going down the level of the quadtree one obtains approximations of various degrees and if one goes down to the bottom most level one obtains the complete image free of any approximation. We have developed an alternative technique to approximate a digitized image which is based on constructing a suitable algebraic operator to do the approximation. We have developed operators for *Erosion* and *Dilation*. We then perform approximations of various degrees by doing a series of dilations and erosions on the given image. We generate a sequence of approximations in this way. The **time complexity** of *erosion* and *dilation* is O(n) where n is the number of terms of the picture polynomial for each iteration.

Thinning. Our thinning algorithm is a specialized erosion processing which deletes from a picture polynomial, at each iteration, those border pixels whose removal does not locally disconnect their neighborhoods. **Time complexity** of each iteration for thinning is O(n) where n is the number of terms of the polynomial. The number of iterations depends on the half of thickness of the object. The algorithm has been coded and implemented on a large number of images and the results are very satisfactory.

We have also developed operators for **magnifying** and **shrinking** of binary images. *Shrinking* followed by *magnification* wipes out the small parts of a digitized image and so it can be used as a **noise cleaning operation.** Our methods give some convenient, easily implementable set of techniques to **identify** and **isolate** *thin parts*, elongated parts, and clustered parts of an image.

We have also considered the problem of efficient data structure for the representation of polynomials corresponding to binary images. We have used the *quadtree* data structure to store and retrieve the polynomials. Some theoretical results have been obtained to study the ratio between the space complexities of our contour determination algorithm when performed on the quadtree representation as compared to its direct application on the polynomial representing the image.

3 OUR WORK ON GRAY IMAGES

The Japanese team of Agui et al considered only binary and colored pictures. We developed methods (including parallel algorithms) to process gray images which extends our work on binary images. Our results obtained so far have been presented recently in several prestigious conferences, see the Refereces at the end, and a journal paper based on this work is currently under review. In our representation of gray images, each polynomial $\sum p_{i,j}X^iY^j$ corresponds a digitized gray image where $p_{i,j}$ is the gray value at the coordinate(i, j); we refer to these polynomials as *picture polynomials*.

First, we define some basic operations on picture polynomials besides the usual operations of addition and multiplication. We then show that most of the standard image processing operations can be done by the basic polynomal operations defined by us. So, it follows that *the polynomial approach is definitely as good as the classical or morphological methods.* In particular, we have developed in detail algorithms for *smoothing, edge detection, morphological operations, dilation and erosion, rotation and coarse magnification.* In all these methods the *time complexity* is $O(n)$ where n is the number of terms in the polynomial representing the gray image. We have used the algebraic

properties of polynomials to great advantage to develop a *template polynomial approach* to image understanding. Work in this direction is in progress and we report here the results obtained so far.

Connected component labeling is one of the basic operations of image understanding. It is analogous to finding the components of a graph. There are several traditional methods to do the image labeling. Here, we apply the polynomial approach to label the connected components. Assume that we have a picture polynimial P representating a binary picture. We have designed a template polynomial for labeling. Given any pixel α corresponding to a term X^iY^j of P, consider the neighbours which are the *right*, *right-down* and *down* neighbours of α. Using the template, we can determine the maximum of the four coefficients of the terms in P coresponding to α and its three neighbours described above. The **time complexity** of our algorithm is propotional to the thickness of the thickest component in the image. For the gray image we usually threshold the image to transform the picture to the binary picture, then do the labeling by the above method.

Shape decomposition is a very important operation in pattern recognition. The algorithm developed by us decomposes a binary shape into a union of simple binary shapes. The decomposition is shown to be unique and invariant to translation and scaling. We call such shape decomposition as a *morphological decomposition* by the polynomial approach. It can be used in pattern recognition and binary image coding. We also see that shape decomposition can also be used to approximate the object.

Template Matching. By cross-correlating a template f with a picture g, we can find a place in the picture g where the template f matches g well. However for a bigger sized template, it will increase the time complexity of computation significantly. In order to reduce the time complexity of computation, we apply the template decomposition to reduce the number of multiplications needed in the computation. We have shown that if the number of terms in the picture polynomial is p, then one needs O(pmn) multiplications in the original computation. By the template decomposition it needs only O(p(m+n)) multiplications. This shows clearly the power of the polynomial approach. The main idea of our parallel algorithm is to decompose or factor the template into products or sum of local templates by polynomial approach, and then to do parallel processing of the image using the local templates. Any seperable template polynomial can be computed locally with respect to the *Von Neumann configuration* by the significantly reduced parallel steps. Since the Von Neumann configuration on an $m \times n$ array simulates mesh connected arrays these methods can be used on such machines. We also obtained an algebraic result using which one can check in a simple manner whether a template can be decomposed.

4 ALGEBRA FOR GRAY IMAGES AND A COLOR ALGEBRA

There has been considerable interest in the past to devise a compact representation of digitized pictures. Ritter *et al* (1987) and (1990), Sternberg (1985) have investigated an algebraic environment, called the *Image Algebra* to optimize, encode and evaluate features of digitized images. In Hwang et al (1989), extensive work has been done for the hardware implementation of the Image Algebra on digital optical cellular logic processors (DOCIP). We extended the algebraic method of representing images (binary, colored and gray) to determine the contour of a binary image, to perform approximation and thinning; also we considered a data structure in the form of a *quadtree* to store and retrieve the polynomials representing the images. Furthermore, we developed a number of algebraic operators for gray and colored images. For colored images, the commutative ring used by us consists of an eight-valued system and we also develop certain algebraic operators for

operations such as the extraction of primary colors, determination of boundaries of various colors. We performed the implementation of our theory for gray images and found that our methods do give satisfactory results within the limitations of our equipment. For colored images, we found that our proposed methods are in agreement with the implementation of our method carried out by Kelley Johnson at the NASA Goddard. The methods proposed by us are very efficient in the sense that they all have *linear time* complexities in the number of pixels.

4.1 GRAY IMAGES

We define a general 2^n-valued algebraic system $A[2^n](\{0,1,...,2^n-1\}, \oplus_{2^n}, \otimes_{2^n})$, in short denoted by $A[2^n]$, for gray image processing. Using this binary-valued system, we develop a number of operations on gray images. Let $GF(2)$ denote the binary field with two elements. In the following definition, we describe two operations $\oplus_{2^n}$ and $\otimes_{2^n}$ for the set of integers $0,1,\ldots,2^n$. Let i and j be any two elements in $\{0,1,\ldots,2^n-1\}$. Represent i and j in the binary representation as $i=(i_{n-1}i_{n-2}\ldots i_0)_2$ and $j=(j_{n-1}j_{n-2}\ldots j_0)_2$. Define $i\oplus_{2^n} j=k$ where $k=(k_{n-1}\ldots k_0)_2$ and $k_r = i_r \oplus_2 j_r$ for r = 0, 1, ..., n-1 and $\oplus_2$ denotes the usual addition in GF(2). Also, define $i\otimes_{2^n} j = s$ where $s=(s_{n-1}\ldots s_0)_2$ and $s_r = i_r \otimes_2 j_r$ for $r=0,1,\ldots,n-1$ and the multiplication $\otimes_2$ is the usual multiplication in GF(2). It can be verified that the algebraic system $A[2^n]$ forms a *commutative ring*. Four *pictorial constants* are defined as follows: P=0: picture whose elements are 0. P=a: picture whose elements are 0 except the pixel at (0,0) which has gray level value a. P=aX: picture whose elements are 0 except the pixel at (1,0) which has gray level value a. P=aY: picture whose elements are 0 except the pixel at (0,1) which has gray level value a. Consider now two $m\times n$ digitized gray images $A=(a_{i,j})$ and $B=(b_{i,j})$ where $a_{i,j}$ and $b_{i,j}$ are the gray levels which take values in the system $A[2^n]$. It is quite standard in digital image processing to use gray levels of number a power of 2. We define two *basic algebraic operations* as follows.

Addition: $C = A\oplus_{2^n} B = (c_{i,j})$ is defined by

$$c_{i,j} = a_{i,j} \oplus_{2^n} b_{i,j} \quad \textit{for all } i,j\ .$$

Multiplication: $D = A\otimes_{2^n} B = (d_{i,j})$ is defined by

$$d_{i,j} = \sum_{p=0}^{i}\sum_{q=0}^{j} a_{p,q}\otimes_{2^n} b_{i-p,j-q}, \quad \textit{for all } i,j\ .$$

Using these two basic algebraic operations, we obtained standard operations on gray pictures. *We shall consider here only 16 gray levels* which is consistent with the digitizer we had at our disposal; our description could be extended easily for any 2^n gray levels and we omit the details. These operations include: **Parallel Movement, Enlargement.** and **Contour of an image.** We have implemented the operators described above on some gray images using an IBM PC/XT and a Hewlett-Packard ScanJet digitizer that could accommodate 16 gray values and found that the results are quite satisfactory. A gray image and its contour obtained by applying our contour operator are shown in FIGURES 1(a-b) respectively. It is feasible to extend our methodology to develop a large number of other standard operations on gray images.

5 COLORED IMAGES

We developed an eight-valued algebraic system to process colored images. Here, the underlying set is $A[2^n]$ with $n=3$ but we shall define the algebraic operations in a different way. As in Agui *et al* (1982b), colors are assigned to the elements of the algebra system A[8] as follows :

0 = black, 1 = blue, 2 = green, 3 = cyan, 4 = red, 5 = magenta, 6 = yellow, 7 = white.

In the above assignment of colors, notice that the *primary colors* blue (B), green (G) and red (R) have the values 1, 2, 4 respectively. It follows that each color is a unique linear combination of these three primary colors and so can be specified by a triple. This color specification system, called in the literature the RGB (Red/Green/Blue) system for further details) is quite traditional. The practitioners of printing technology have found it convenient to define the arithmetic rules among colors according to the scheme given in Agui *et al* (1982b). From this table one sees that the operations of addition and multiplication as defined, do not satisfy the axioms of a ring. Our objective had been to give the system an algebraic formulation such that the underlying operations would hopefully conform to the axioms of a suitable algebraic structure, for example that of a commutative ring; further the operations would be expected to correspond to the usual operations on colors. First, we formulated a new algebraic system defined which has the operations $\oplus_8$ and $\otimes_8$. We see that this system satisfies the conditions of a ring but unfortunately, it does not correspond to the usual operations among colors. Based on the algebraic system we consequently proposed another algebraic system where the addition $+_8$ resembles more closely the real arithmetic operation in colors. Thus we have developed an algebraic system for colored images which we call the *color algebra*. For a color picture A, we can get its twice enlarged image B as follows: $B = A \otimes_8 A$. Since $a \oplus_8 a = 0$ *and* $a \otimes_8 a = a$, one sees that B is a *coarse enlargement* of A. If we want to get a *dense enlargement*, we can use the following operation:

$$B = (A \otimes_8 A) \otimes_8 (7 \oplus_8 7X) \otimes_8 (7 \oplus_8 7Y).$$

Similarly, we can also get a 2^n-enlarged picture from the original one by an analogous procedure.

Extraction of the primary colors

Taking advantage of the property of multiplication $\otimes_8$, we can extract the primary colors from a color picture. Only the primary color blue will be extracted if any color picture is multiplied by 1. Also, only the green or red primary colors will be extracted if the color picture is multiplied by 2 or 4. We also can make a multiple arrangement of a color picture A by a multiplication of A and B in which there are 7 different color pixels which separate far enough so that the result of the multiplication will be a picture including 7 separate different color multiple arrangements.

Color Contour

Given a color picture A, in the following equations we define a new picture B which is composed of the boundaries of each color region of a given picture A. For example, the boundary in the X direction is given by:

$$B_X = A \otimes_8 (7 \oplus_8 7X).$$

Also, we can get the *primary color boundary* picture from A for the various primary colors. For example, the blue boundary in the XY direction is given by:

$$B_{XY} = A \otimes_8 (1 \oplus_8 XY).$$

Color Contour Oprator. Using an operator similar to the contour operator for gray images, we can design the *color contour operator* as follows. For a color picture A, the *color contour* C is given by: $C = A *_8 U \oplus_8 A$ where U is the template polynomial corresponding to an 8-connected neighbors.

We developed an algebraic environment to process gray and colored images. The operations proposed here run on linear time in the number of pixels and so are efficient. From the implementations carried out by us for gray images and from the implementation for colored images carried out at the NASA Goddard by Kelley Johnson, we see that the experimental results agree satisfactorily with the theory developed. It would be interesting to develop further image recognition methods at the pixel level using the algebraic approach introduced in this paper. Further, it would be of interest to develop parallel algorithms based on our approach for processing images.

6 3D IMAGE UNDERSTANDING

Introduction

We extended the polynomial approach which has been done so far for representing 2-D images only, to 3-D vision and we develop a number of algorithms for image understanding. All our algorithms have *linear time* complexity.

The *volume representation* of a 3-D object is described as follows. First, the object is approximated by a volume occupancy array, each cell of the array is usually referrred to as a *voxel*. Our *polynomial representation* of the 3-D object is then given as follows. Given any 3-D object A, we shall represent it by a polynomial in 3 variables: $A \equiv \sum_{i,j,k} p_{ijk} X^i Y^j Z^k$ where $p_{ijk} = 1$ if the voxel (i,j,k) of the array lies completely inside A and otherwise we set $p_{ijk} = 0$. We shall refer to such a polynomial as the *picture polynomial* of the given 3-D object.

3-D Vision Algorithms

The polynomial representation can provide a convenient environment for certain transformations on a 3-D object, like rotation, translation and magnification. These operations are very useful in 3D Object recognition which is of significant interest to the AFOSR.

Consider a 3-D object represented by a polynomial $P(X,Y,Z)$. Affine transformation can be done by replacing X, Y and Z by the following: $X \rightarrow X^i Y^j Z^k$, $Y \rightarrow X^l Y^m Z^n$ $Z \rightarrow X^a Y^b X^c$. So, the polynomial $P(X,Y,Z)$ is transformed into $P(X^i Y^j Z^k, X^l Y^m Z^n, X^a Y^b Z^c)$.

Rotation of 90 degrees about Z axis: Let $i = 0, j = 1, k = 0$; then, $P(Y, X^{-1}, Z)$ gives the polynomial representation of the object rotated by 90 degrees about the Z axis from the original object represented by $P(X,Y,Z)$. Similarly, one can also consider rotations of multiples of a right angle about the other co-ordinate axes.

Morphological Operations By Polynomial Approach

The primitive operations of Mathematical Morphology are *dilation* and *erosion* by means of a *structuring element* which could be a standard shape, for example, a square, hexagon or a circle. Let X denote a binary image and B be a structuring element. The *Minkowski operations* of addition $\oplus$ and subtraction $\ominus$ are defined as follows.

$$X \oplus B = \{x : B_x \cap X \neq \emptyset\}, \quad X \ominus B = \{x : B_x \subset X\}$$

where B_x denotes the structuring element positioned with its locus of centers at x. Now, the *dilation* and *erosion* are described by

$$X \oplus B = \bigcup_{x \in X} B_x, \quad X \ominus B = \bigcap_{b \in B} X_b$$

respectively; erosion is the dual of the dilation operation. In 3-D the *structuring element* could be a solid of a specifically known shape, for example, a cube, cylinder, tetrahedron, sphere etc., all of which of course being considered in the forms approximated by voxels. We take a *structuring element* (SE) and slide it over or under the surface of a given 3-D object. *Erosion* of a 3-D object X by a 3-D structuring element B is found by positioning B everywhere under the surface such that B is contained in X. The set of the surviving voxels of B is called the *erosion* of X by B. The *dilation* of X by B is found by placing B everywhere on the surface of X such that B contains at least one cell in X. The set of all voxels of B which is contained in X is called the *dilation* of X by B. We showed how the morphological operations can be carried out conveniently using the polynomial representation. Let $X \equiv \sum_{(i,j,k)\in X} X^iY^jZ^k$ and $B \equiv \sum_{(i,j,k)\in B} X^iY^jZ^k$ be the picture polynomials of a 3-D object and a *structuring element* (SE) respectively.

Determination of the Surface Suppose A is the picture polynomial of a 3-D object. Let T be the picture polynomial of the SE. The following operator would give the *surface* (contour) of A : $(A \bigoplus T) - A$ where $\oplus$ denotes the dilation operation defined in (1) and the $-$ denotes the usual subtraction of polynomials. It is now possible to write a program using (2) for contour determination. Figure 1 shows a 3-D object and its surface obtained by implementing this method by taking SE as a *ball*.

Shape Decomposition One way to recognize the features of a 3-D object is to decompose it as a union of certain known simple shapes *(primitives)* $P_1, P_2, ..., P_k$. Suppose that the simple subsets P_i are chosen as 3-D objects which are of similar shape but of various sizes, for convenience, we shall refer to these as *balls*. We choose the kind of *ball* we wish to use for the decomposition. First, the set X_1 of maximal inscribable balls in an object that have the maximum radius, is found. This set is the *first cluster* of the decomposition. The second set is obtained as follows. The first set of the decomposition is subtracted from the object. Then, the set of maximal inscribable balls in the remaining of the object that have the maximal girth, is found. This set is the *second cluster* of the decomposition. The first and second set of the maximal inscribable balls is subtracted from the object and the procedure is repeated until the remaining is an empty set. First, we use the *erosion* procedure to find L_1 by successive erosions of the original solid object X by the structure element B until the result of the erosion becomes an empty set at the $(N_1 + 1)$th step of the execution of the algorithm where N_1 is the size of the maximal object inscribable in X. Now L_1 and N_1 are stored in a file. The *dilation* procedure calculates X_1 by N_1 successive dilations of L_1 by B. The set-theoretic difference $X \setminus X_1$ is calculated and is stored in X. This procedure is repeated several times until the size N_1 of the maximal inscribable object in $X \setminus X_{i-1}$ given by erosion procedure is equal to 1. This is the *stopping condition* for the algorithm. So, an object can be represented by a union of simple objects $X_1, ..., X_k$. Each of these objects is completely described by the locus L_i and the girth r_i of the corresponding SE's. In certain cases the locus L_i is composed of a disconnected subset of $L_{i1}, ..., L_{ik}$ which corresponds to disconnected subsets $X_{i1}, ..., X_{ik}$ of the simple object X_i. Suppose P_0 is the original 3-D picture polynomial and P_d is the shape decomposed picture polynomial.

7 LIST OF PROFESSIONAL PERSONNEL ASSOCIATED WITH THE PROJECT

- Dr. Prabir Bhattacharya, the Principal Investigator. He spent one summer month during the summer of 1990 and also spent 25 percent of his working hours during the academic year.

- Kai Qian was the research assistant in this project from the start till May, 1990 when he got

his Ph.D. based on the efforts done in connection with the project.

Degree awarded: Ph.D.

Date of award: May, 1990.

Thesis title: "Image Processing and Pattern Recognition by Polynomial Approach"

Institution: University of Nebraska-Lincoln, Department of Computer Science & Engineering.

- Lu Xun who became the research assistant after Kai Qian had graduated. (His appointment was santioned by Dr. Waksman, the Program Manager.) Mr. Xun successfully completed the defense of a MS thesis in August, 1991.

  Title of thesis: "New Fast Algorithms for Thinning of binary images".

8 OUTPUT

8.1 Journal Papers

1. K. Qian and P. Bhattacharya, Binary Image Processing by Polynomial Approach, *Pattern Recognition Letters*, (Publisher: North-Holland), Vol. 11, pp. 395-403 (1990).
2. K. Qian and P. Bhattacharya, A template polynomial approach for for image processing and visual recognition. Submitted for publication to the *Pattern Recognition* and is under review.
3. K. Qian and P. Bhattacharya, An Algebraic System for Processing Colored Images, *Pattern Recognition Letters*, accepted for publication.
4. K. Qian and P. Bhattacharya, Image Processing By Polynomial Approach, *Jour. of Computing & Information*, Vol. 1, pp. 69-78 (1990). (Publisher: Inst. Gaussianum, Berlin).
5. K. Qian and P. Bhattacharya, Determining Holes and Connectivity in Binary Images, under review in *Internat. Jour. of Mathematical Imaging* (Publihser: Kluwer).
6. P. Bhattacharya and K. Qian, A 3D object recognition system using the voxel recognition, under preparation and to be submitted by September, 1991.
7. X. Lu and P. Bhattacharya, New, fast parallel thinning algorithms and skeletonization, under preparation and to be submitted by December, 1991.

8.2 Papers published in conference proceedings

1. K. Qian and P. Bhattacharya, A Polynomial Approach To Image Processing and Quadtrees, *Proc. IEEE Internat. Conf. Computers and Communications, Arizona, 1989*, pp. 596-600.
2. P. Bhattacharya and K. Qian, A Polynomial Algebra Approach to Binary and Gray Image Processing, Proc. of the Internat. Conf. on Aerospace Pattern Recognition, SPIE vol. 1098 (1989), pp. 110-121.
3. Kai Qian and P. Bhattacharya, Contour Determination and Approximation of Binary Images by Polynomial Operators, Proc. of Internat. Sympos. on Digital Image Processing, San Diego, CA, 1989, Vol. 1153 (Published in Jan., 1990), pp. 668-674.

4. P. Bhattacharya and K. Qian, A Polynomial Image Approach for Image Processing, Proc. of the Conf. on Visual Communications and Image Processing, Philadelphia, 1989, SPIE vol. 1199, pp. 286-295.
5. P. Bhattacharya and K. Qian, A Parallel Algorithm for Template Matching, Proc. of IEEE Internat. Conf. on Computers & Communication, Phoenix, AZ, March, 1990. p. 862.
6. P. Bhattacharya and K. Qian, Parallel algorithm for skeletonization of binary and gray images, Internat. Sympos. on Artificial Intelligence and Mathematics, Fort Lauderdale, FL, Jan., 1990. Abstract.
7. P. Bhattacharya and K. Qian, Software Development: Image Processing By Template Polynomials, Proc. of Annual Conf. of the ACM, Feb., 1990, Washington, D.C., pp. 276-280.
8. P. Bhattacharya and K. Qian, Image processing By Template Polynomials, Conf. on Applications of Artificial Intelligence, Orlando, FL, 1990, SPIE vol. 1293, pp. 549-556.
9. P. Bhattacharya and K. Qian, Compact Template Polynomials Methodology in Pattern Recognition and Parallel Processing, IEEE Internat. Sympos. on Circuits & Systems, New Orleans, LA, May, 1990 , pp. 1485-1488.
10. P. Bhattacharya and K. Qian, A Polynomial Approach for Morphological Operations on 2-D and 3-D Images, *Proc. of the SPIE Internat. Sympos. on Intelligent Robots and Computer Vision*, Boston, MA, Nov., 1990, pp. 530-536.
11. P. Bhattacharya and K. Qian, Geometric Property Measurement of Binary Images By Polynomial Representation. *Internat. Sympos. on Optical Engineering: Applications of of Artificial Intelligence* IX, Orlando, Florida, April, 1991. Published as a full-length paper in the proceedings.
12. P. Bhattacharya and K. Qian, Some New Parallel Algorithms for Thinning Binary Images, *Internat. Sympos. on Optical Engineering: Applications of Artificial Intelligence* IX, Orlando, Florida, April, 1991. Published as a full-length paper in the proceedings.
13. P. Bhattacharya and K. Qian, Polynomial Approach for Developing Vision Systems, *IEEE Internat. Conf. on Systems, Man and Cybernetics*, October, 1991, Charlottesville, VA, accepted to appear in the Conf. Proc..
14. P. Bhattacharya and X. Lu, Some fast, parallel algorithms for thinning of binary images *IEEE Internat. Conf. on Systems, Man and Cybernetics*, October, 1991, Charlottesville, VA, accepted to appear in the Conf. Proc.

8.3 Invited addresses

1. The Principal Investigator gave an half-hour invited address at the **Eighth International Congress on Cybernetics & Systems**, held in Manhattan, N.Y. in June, 1990. In this address he described the work being done by him and his students in this project supported by the AFOSR. An Abstract of the address appeared in the Congress Proceedings.

8.4 Consultative and advisory functions to other laboratories

a) The Principal Investigator visited the **Center for Automation Research, University of Maryland**, College Park several times during 1989-91 for short periods and collaborated in research with Prof. Azriel Rosenfeld.

b) The P.I. has visited the **NASA Goddard Space Flight Center**, Greenbelt, MD and discussed his work. Furthermore, he gave two presentations there in July 1990 and November, 1989. His work on colored images (now to appear in the *Pattern Recognition Letters*) was implemented by an engineer at NASA Robotics Lab who wrote an internal technical report about it as a part of her Professional Improvement Program.

c) Th work of the P.I. on 3D Voxel representation has been of interest to researchers at the **European Molecular Biology Institute**, Heidelberg, West Germany. I have been contacted by Dr. D. Wild of that Institute who expressed that my work had significant potential applications in the determination of protein structures. He recently implemented my 3D voxel representation algorithms described in section 6. A possibility of collaboration between a group at that Institute and the Principal Investigator is being currently discussed.

d) The P.I. is an associate member of the *Center for Communication & Information Sciences at the University of Nebraska-Lincoln.* This Center has provided matching funds to augment this Grant and has taken an interest in this project.

9 CONCLUSION

We have found that the image processing methodology using the *picture polynomials* is an efficient, interesting and fruitful approach offering a large number of possible techniques to understand a digitized image. Further, the image manipulations can be performed, described and understood very conviniently since polynomials are very easy to manipulate and familiar to everybody. We are actively engaged in developing and implementing further methods based on this approach. Our future plan is to develop a 3D Object Recognition System based on the polynomial approach.

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